

The Role of Shock Induced Trailing-Edge Separation In Limit Cycle Oscillations*

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In figure 1 Limit Cycle Oscillations (LCO) are defined as limited amplitude oscillations which are self-sustaining and are produced by a structural/aerodynamic interaction. The role of shock induced trailing edge separation (SITES) in this phenomenon, as will be discussed in this paper, is to act as a non-linear spring which triggers and drives the LCO. The appearance of SITES coincides with the classical trailing edge pressure divergence which is a well-known indicator of buffet onset. Because of this, LCO has previously been referred to as buffet or, at transonic speeds, Mach buffet. The conditions for transonic LCO are moderate incidence, usually less than 10° , and high transonic Mach numbers, ranging from 0.8 to 1.1. Lowly damped vibration modes tend to respond provided they have the proper characteristics to couple with the SITES type flow. These conditions frequently occur near flutter boundaries which creates considerable anxiety in both test pilots and engineers, especially when the response is in a single mode with a distinct frequency. Since amplitudes can become quite large, even though they are limited, the knowledge of LCO boundaries becomes very important for efficient flight flutter testing.

- **LCO Is Defined As A Limited Amplitude Oscillation Which Is Self-Sustaining And Results From A Structural/Aerodynamic Interaction**
- **Shock Induced Trailing Edge Separation (SITES) Coincides With The Classical Trailing Edge Pressure Divergence**
- **The Conditions Are Moderate Incidence And High Transonic Mach Numbers**
- **Lowly Damped Vibration Modes Tend To Respond If They Have The Proper Characteristics**
- **Response Is Single Mode With Distinct Frequency And Limited Amplitude**

Figure 1

WING BENDING AND TORSION MODE RESPONSE FROM THE F-111 TACT BUFFET ANALYSIS AT $M=0.8$, $\Lambda_{LE}=26$ DEG

The results shown in figure 2 represent the comparison of predictions and flight test results for buffet response of the F-111 tact aircraft at $M=0.8$ and a leading edge sweep of 26° (ref. 1). The results are wing tip RMS accelerations for the first symmetric wing bending mode and the first wing torsion mode group which includes both asymmetric as well as symmetric modes. Several altitudes are represented which range from 13K ft to 28K ft, for angles varying from 7° to 12° . The interesting feature to note in the comparisons is that (1) the bending mode responses are very well predicted in terms of both angle and altitude effects whereas (2) the torsion mode responses are consistently underpredicted with exception of the lowest altitude. Since the prediction method did not allow for a coupling between the buffeting flow and the structural response, these results led to an investigation of what kind of coupling could exist with the torsion modes. Thus, the role of SITES in LCO was developed as an answer to this question.

(Reference 1)

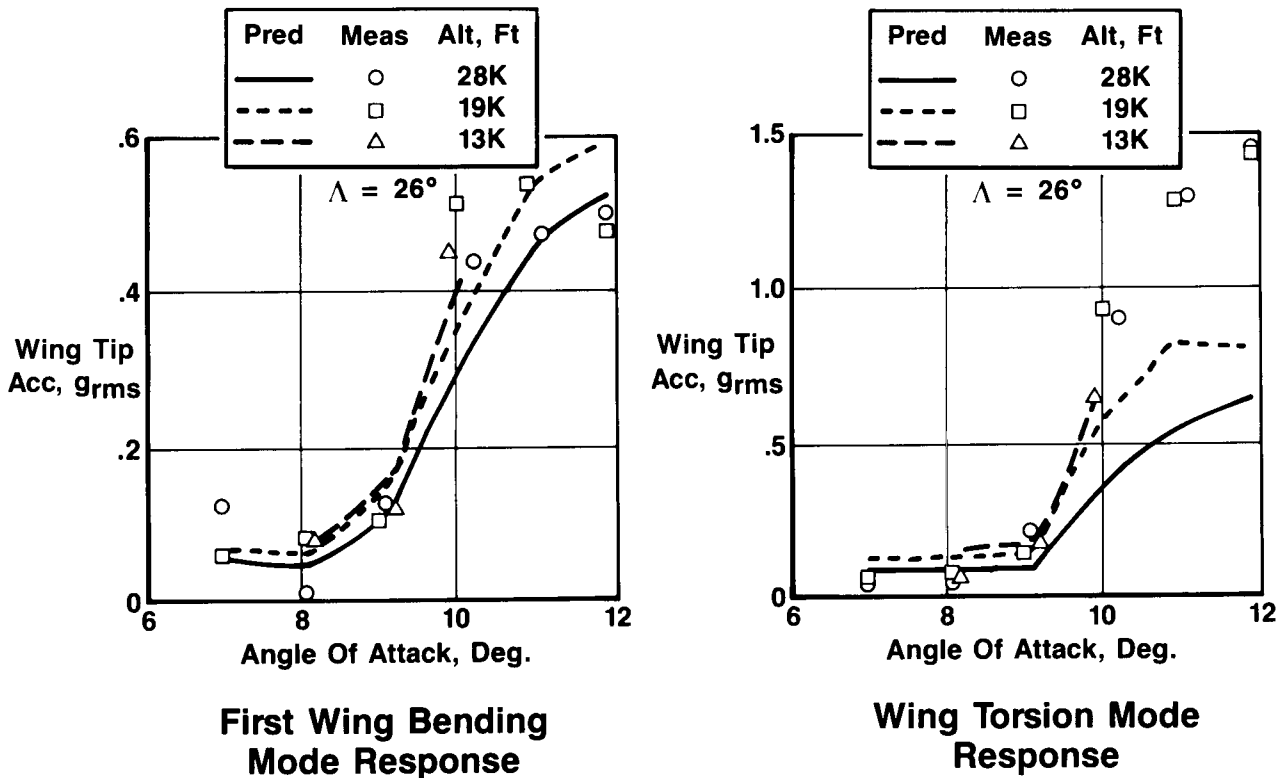
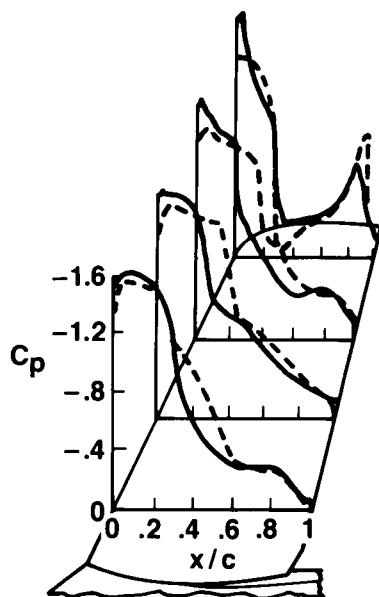


Figure 2

STATIC PRESSURE CHANGES ON THE MODEL AND AIRPLANE WING UPPER SURFACE FOR TRANSITION TO SITES

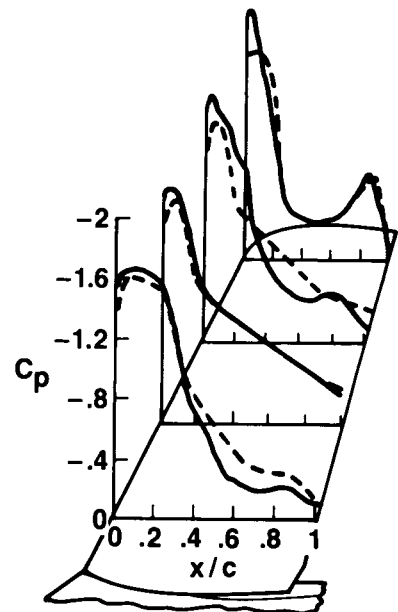
A clue to a possible means for the coupling can be deduced from the static pressures in figure 3. The occurrence of trailing edge pressure divergence at about 10° also corresponds to a large forward movement of the upper surface main shock as shown by comparing the pressures in figure 3 at 9° and at 10° . It will also be noted that the forward shock movement for the aircraft is much larger than that for the model. The condition of this transition is the occurrence of shock-induced trailing edge separation (SITES) which was extensively discussed by Cunningham, et al in reference 2. It was shown that this transition was accompanied by a step change in pitching moment with either increasing or decreasing angle-of-attack. With increasing angle, the forward shock movement produced a loss in lift forward and the trailing edge divergence produced a gain in lift aft. The net result was to provide a step change in pitching moment that was nose down for increasing angle. For decreasing angle, the opposite took place and produced a step change in pitching moment that was nose up.

(Reference 1)



$\alpha = 9.1$ Deg
Pre-SITES

— 1/6 Scale
Model
 $R_{MAC} = 14 \times 10^6$
--- FLT Test
 $R_{MAC} = 25 \times 10^6$
 $M = 0.8$
 $\Lambda_{LE} = 26$ Deg.



$\alpha = 10$ Deg
Post-SITES

Figure 3

EFFECT OF MEAN ANGLE, FREQUENCY AND AMPLITUDE ON MEASURED UNSTEADY PRESSURES

Dynamic unsteady investigations of SITES flows were conducted by Triebstein (ref. 3) for an NACA 0012 airfoil oscillating in pitch at various mean angles, frequencies and amplitudes as shown in figure 4. These results are the upper surface unsteady pressure distributions for the first harmonic. The effect of varying mean angle is quite pronounced where at $\alpha = 0^\circ$ the distributions are of conventional transonic characteristics, but at $\alpha = 5^\circ$ the distributions are totally different. The most notable change is the trend toward a more nose-down pitching moment. The influence of increasing frequency simply shows an expected increasing lag at either angle. Increasing amplitude at $\alpha = 5^\circ$ with SITES shows a decrease of the shock motion peak amplitude when normalized by the pitching amplitude. This demonstrates the tendency to maintain a constant incremental force with increasing pitch increments so that a limited amplitude motion is inevitable. That is, the viscous damping which is a function of amplitude increases until it balances the destabilizing fixed incremental force due to the SITES transition.

(Reference 2)

Upper Surface Results For An Oscillating NACA 0012 Airfoil, $M = 0.78$

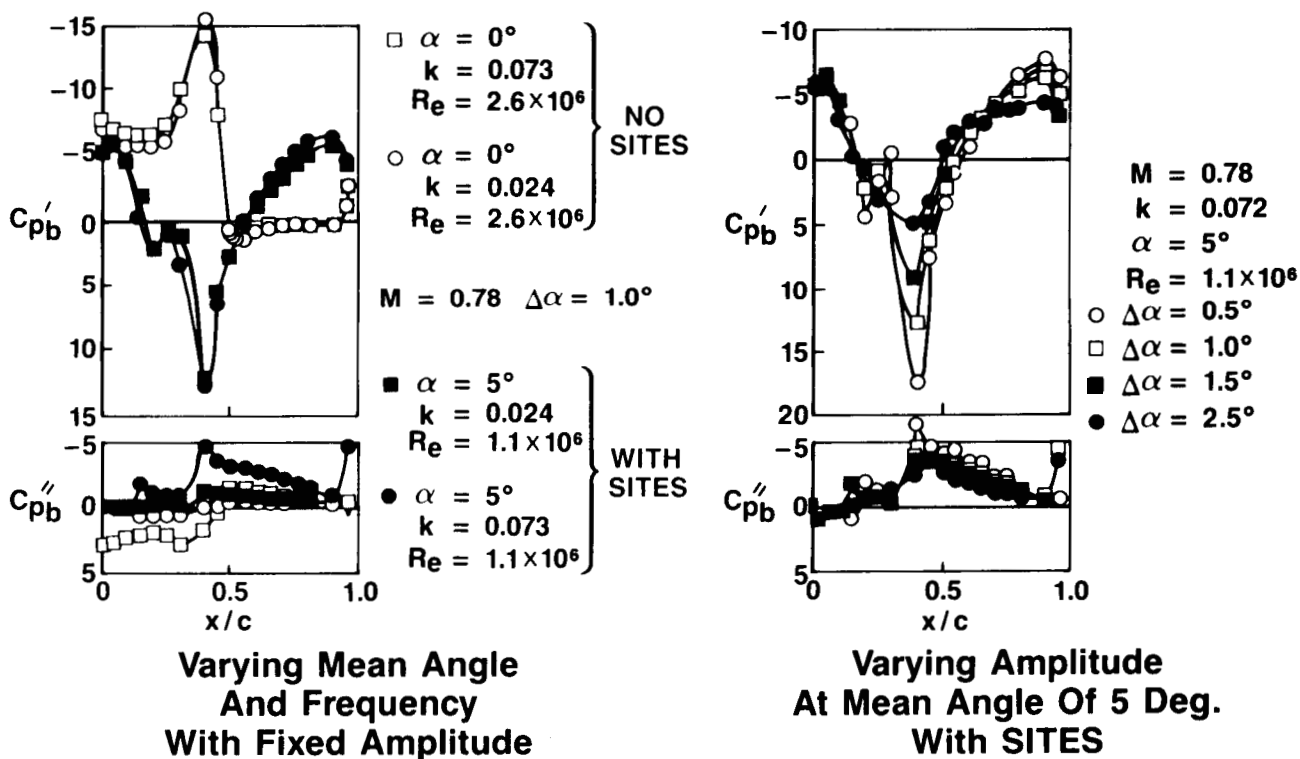


Figure 4

SCHEMATIC DIAGRAM OF THE VARIATION OF THE INCREMENTAL PITCHING MOMENT DUE TO SITES WITH ANGLE AND PITCH RATE

The step change in pitching moment just described in figures 3 and 4 can be cast in the form of a non linear spring that provides a step increase in resisting stiffness as the airfoil passes through SITES. This form is illustrated in figure 5 for a hypothetical situation in which, for simplicity, only the step change is present. (Normally, the pitching moment variation with angle is a nearly constant slope with the discontinuity superimposed to produce a shift in characteristic at the SITES transition point.) The influence of pitch rate that provides the hysteresis is also illustrated. For positive pitch rate, SITES is delayed to a higher angle, whereas for negative pitch rate, re-attachment is delayed to a lower angle.

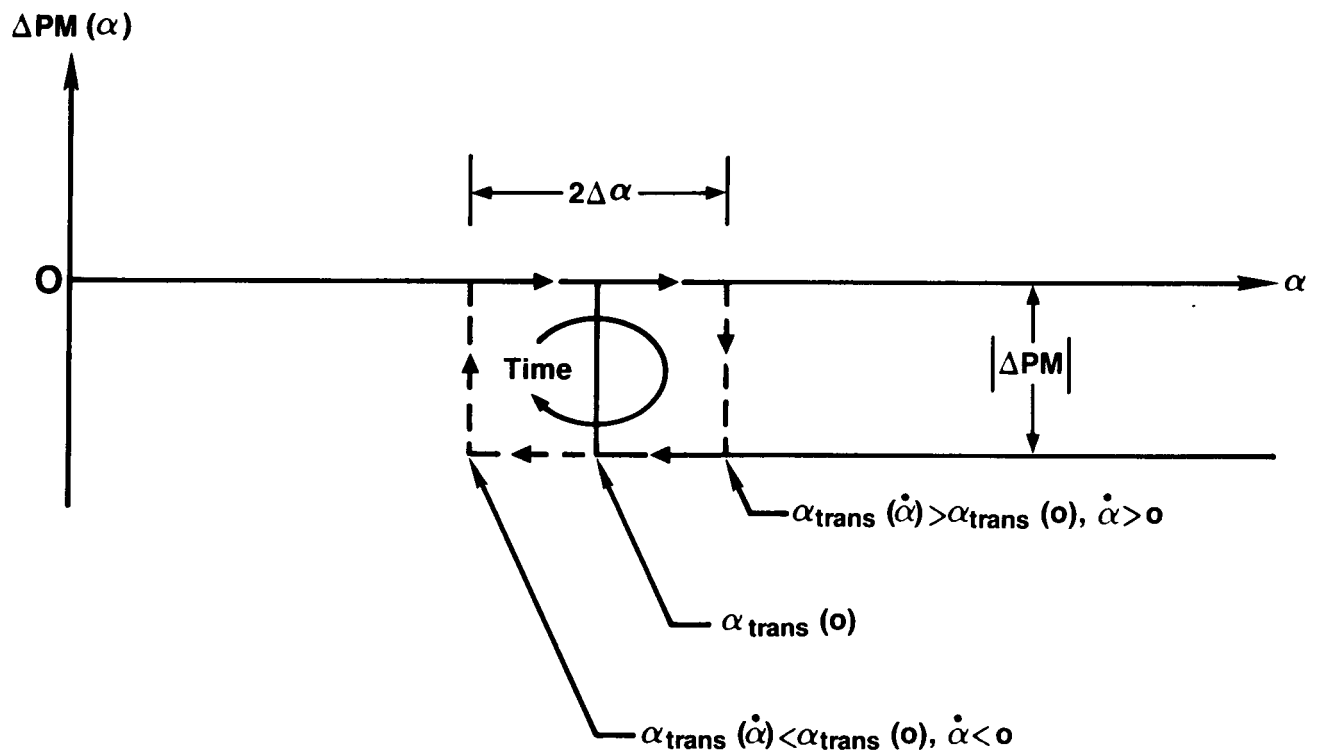


Figure 5

HYPOTHETICAL TIME HISTORY OF AN AIRFOIL UNDERGOING SITES INDUCED TORSIONAL LCO

The non-linear spring described above can potentially produce a limit amplitude, self-sustaining oscillation. How this is possible can be described by considering an airfoil with a torsion spring undergoing a maneuver of increasing angle until it encounters SITES as shown in figure 6. Slowly increasing incidence at angles below that of SITES allows the torsion spring to attain a continuous state of equilibrium with aerodynamic pitching moment. When SITES is reached, a sudden nose-down increment is imposed on the aerodynamic pitching moment which will tend to reduce wing incidence. This will be a dynamic negative pitch rate which will delay re-attachment and permit the nose-down moment to put work into the system. At some point, re-attachment does take place and the nose-down moment disappears. Accelerations become negative and the wing experiences a reduced pitch rate until it reaches zero and begins nose-up motion. Positive pitch rate now takes over which will produce a delay in SITES and allows an overshoot of the initial starting point due to stored elastic energy during the down stroke. When SITES does occur, the cycle then repeats itself.

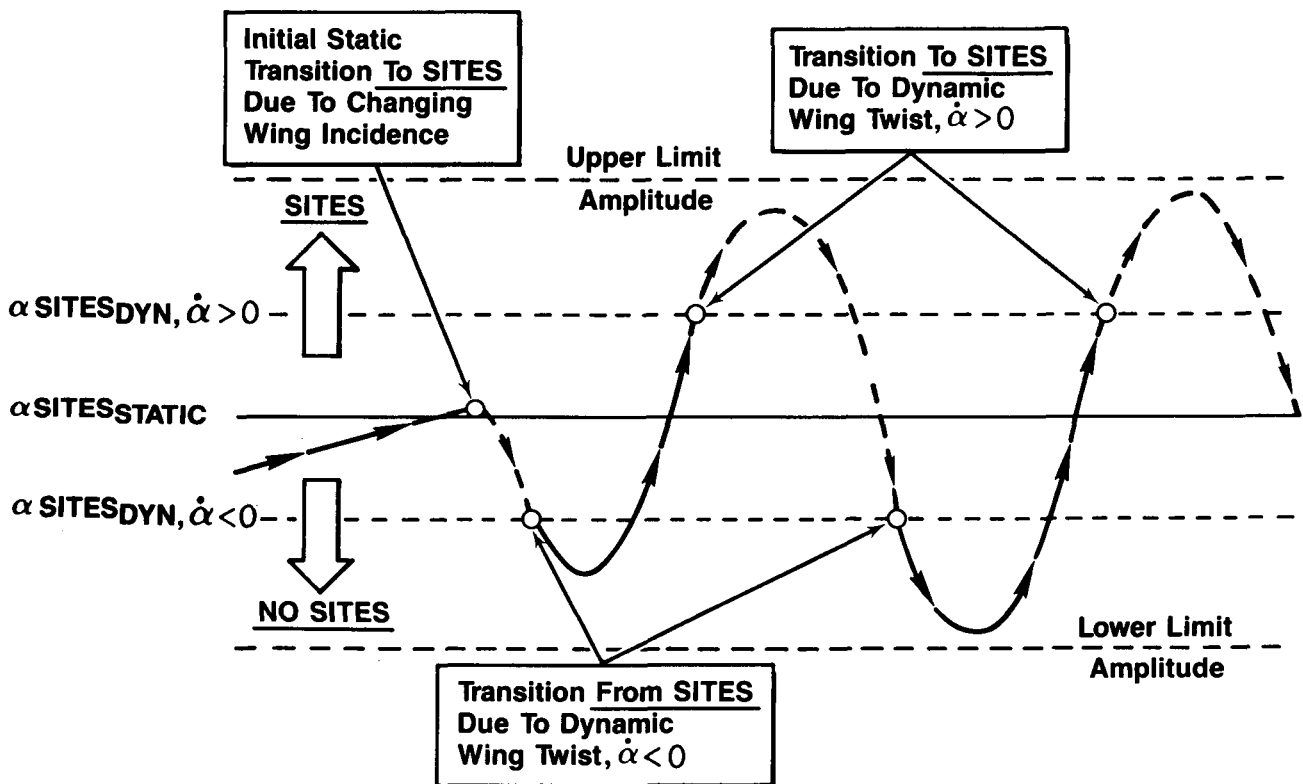


Figure 6

A MATH MODEL FOR LCO

The math model for calculating LCO response was developed and presented in reference 1. The governing equation for a non-linear single DOF spring-mass system is shown in figure 7. The LHS of the equation is the conventional linear equation form for a generalized coordinate response. The RHS is composed of the non-linear step force function as defined for the vibration mode of interest. This force is obtained by integrating the incremental pressure changes due to SITES transition with the mode shape to produce a generalized force. The boundary conditions require that at time zero, the system is in static elastic equilibrium (zero velocity) and that the normalized displacement is equal to zero at the static transition point for SITES. That is, the step function change on the RHS is suddenly imposed on the system at time zero.

- **The Non-Linear Single DOF Spring-Mass System**

$$M_i \ddot{q} + 2 M_i \omega_i \delta_i \dot{q} + M_i \omega_i^2 q = \Delta F_i(q, \dot{q})$$

- **Where**

$M_i, \omega_i, \delta_i, q$ = Mass, Frequency, Damping and Response
For The i th Mode

$\Delta F_i(q, \dot{q})$ = Non-Linear Incremental Force
(Due To SITES)

- **Is Subject To The Initial Conditions**

$$q(t = 0) = q_{\text{trans}} (\dot{q} = 0)$$

$$\dot{q}(t = 0) = 0$$

Figure 7

SCHEMATIC DIAGRAM OF THE VARIATION
OF $\Delta F_i(q, \dot{q})$ NEAR SITES

The plot shown in figure 8 illustrates that the form of variation of the step change in generalized force is identical to that shown for pitching moment in figure 5. The influence of pitch rate in the hysteresis is also identical for wing modes that have significant torsional motion. This is consistent with observations to date which indicate that LCO occurrences which are coincident with SITES tend to concentrate on torsional modes or modes with some pitching content.

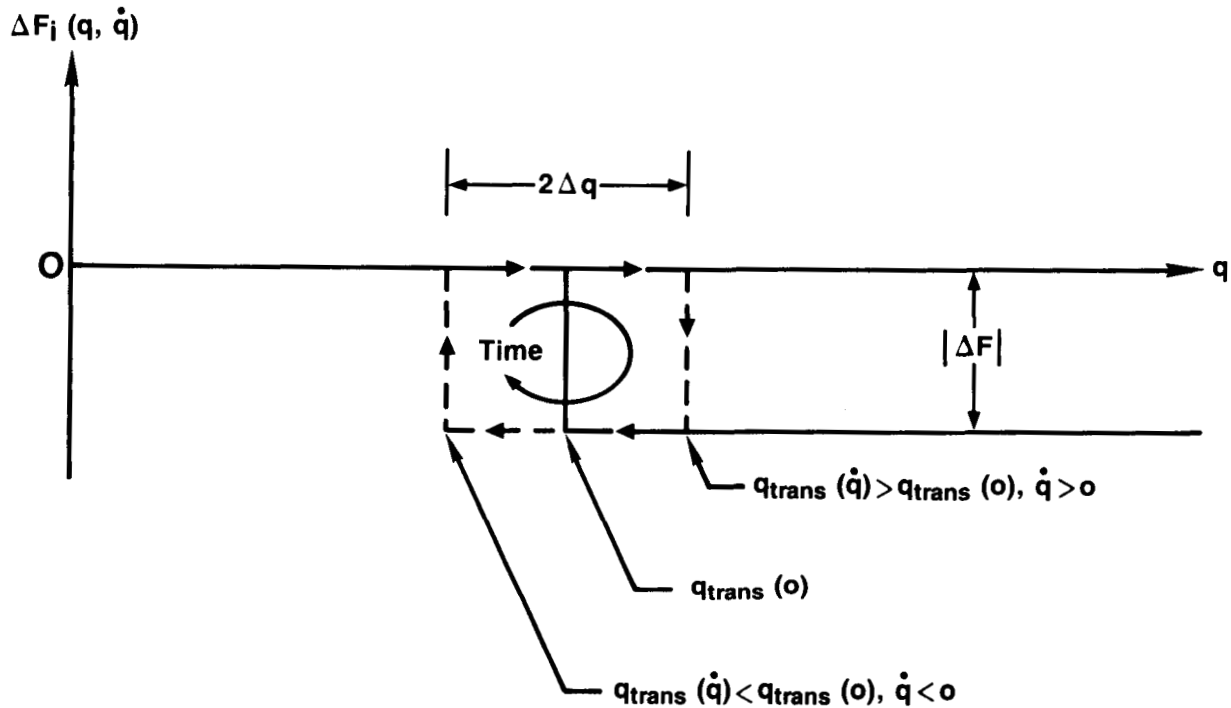


Figure 8

A MATH MODEL FOR LCO (contd.)

A finite difference solution was developed in reference 1 for the equation of motion discussed in figure 7. The form of the solution is shown in figure 9 where the parameters have all been cast in non-dimensional form with exception of ϵ and q_n . These terms have the dimension of length. The two equations are applicable depending on the response value relative to SITES transition. Hysteresis is included in the model through the use of a transition point that is sensitive to wing motion velocity, \dot{q} . The initial conditions are defined such that the transition actually occurs at the third time step, q_2 .

- **A Finite Difference Solution To The Non-Linear Equation Is**

$$q_{n+1} = \frac{1}{1+\lambda\delta} [\lambda^2\epsilon + (2 - \lambda^2)q_n - (1 - \lambda\delta)q_{n-1}], \quad q_n \geq q_{\text{Trans}}(\dot{q})$$

$$q_{n+1} = \frac{1}{1+\lambda\delta} [(2 - \lambda^2)q_n - (1 - \lambda\delta)q_{n-1}], \quad q_n < q_{\text{Trans}}(\dot{q})$$

- **Where**

$$\lambda = \Delta t \omega_i \qquad \epsilon = \frac{\Delta F_i}{M_i \omega_i^2}$$

$$\delta = \delta_i$$

Δt = Time Step Size

q_n = Response Of The i th Mode At The n th Time Step

- **Subject To The Initial Conditions**

$$q_0 = q_1 = 0, \quad q_2 = \frac{\lambda^2\epsilon}{1 + \lambda\delta}$$

Figure 9

SAMPLE OF LCO CALCULATIONS FOR THE F-111 TACT

The algorithm just described was programmed and input data were developed for the right wing torsion mode of the F-111 TACT airplane (ref 1). The conditions were $M=0.8$ and 21K ft altitude. The generalized mass, frequency and damping data were obtained from a conventional dynamic response and flutter analysis. The estimate for ΔF_i was developed by using the pressure distributions at 9° and 10° to obtain the incremental loads due to transition to SITES. The load distribution was combined with the mode deflections, $h_i(x,y)$ to produce the generalized step force value of ϵ . Estimates for Δq , the hysteresis parameter (see fig. 8), were not possible based on available data, hence, a parameter study was conducted by letting Δq vary as a fraction of ϵ . A sample plot for $\Delta q = \epsilon$ shown in figure 10 illustrates how the transient solution quickly approaches a limit amplitude oscillatory motion. It is also interesting to note that the apparent frequency is slightly higher than the right wing torsion mode frequency. (Since the number of time steps shown in the plot represents ten cycles, the apparent frequency can be determined by simply counting cycles.)

• The Parameters For The Right Wing Torsion Mode Were

$$M_i = 565.9 \text{ Lbs}$$

$$\omega_i = (14.17 \text{ Hz}) \times 2\pi$$

$$\delta_i = 0.07 \text{ (From Flutter Solution)}$$

$$\begin{aligned} \epsilon_i &= \frac{Q}{M_i \omega_i^2} \int_{A_{\text{wing}}} h_i(x, y) [C_{p_{10}}(x, y) - C_{p_9}(x, y)] dx dy \\ &= -0.0127 \text{ Ft} \end{aligned}$$

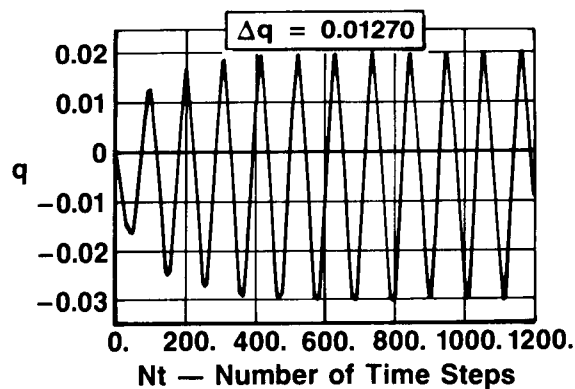


Figure 10

RESULTS OF LCO CALCULATIONS FOR THE F-111 TACT AIRCRAFT AND 1/6-SCALE MODEL RIGHT WING TORSION MODE

The results shown in figure 11 are for the nominal conditions of $M=0.8$, $\alpha=9^\circ-10^\circ$ as were presented in reference 1. The flight test data were extracted from figure 2 by subtracting the predicted response from the flight measured response for the wing torsion modes. Since the LCO attributed response was assumed to be uncorrelated with the broad-band buffet response, this subtraction was done on an RMS basis. The calculated full scale data were obtained with the LCO algorithm by letting Δq vary as a fraction of ϵ from 0.2 up to 2.0. It is interesting to note that up to a ratio of 1.0, the LCO response increases and the frequency drops. Within this range, the calculated amplitude is within reasonable agreement with the flight data considering that the method should be conservative as it currently exists. For $\Delta q = 2\epsilon$, the response is zero, which states that the conditions for sustained oscillation require that the transition must occur in both directions during the cycles; otherwise, the response is simply a static displacement. Finally, the 1/6-scale model results show that the calculations verify the observation that the model did not experience LCO in the wind tunnel test.

Conf.	Method	ϵ (Ft)	$ \epsilon$ (Deg.)	$ \Delta q/\epsilon $	APP. f, Hz	RWAC. g's Response*
F.S.A.C.	FLT Test	—	—	—	15.0	0.8
F.S.A.C.	Calc.	-0.0127	0.073	0.2	18.4	0.71
				0.4	17.0	1.42
				0.6	16.4	1.79
				0.8	15.1	2.05
				1.0	14.9	2.34
				2.0	14.2	0
1/6-S.M.	Wind Tun.	—	—	—	156.0	≈ 0
1/6-S.M.	Calc.	-0.000253	0.0087	8.4**	156.0	0

*LCO Response From Testing Is Estimated As The Amount That Exceeds Expected Linear Buffet Response

** Δq Assumed As A Constant Angle Determined By F.S.A.C. For $|\Delta q/\epsilon| = 1.0$

Figure 11

CONCLUDING REMARKS

As a result of this investigation, the potential role of shock induced trailing edge separation in limit cycle oscillations was established. It was shown that the flip-flop characteristic of transition to and from SITES as well as its hysteresis could couple with wing modes with torsional motion and low damping. This connection led to the formulation of a very simple non-linear math model using the linear equations of motion with a non-linear step forcing function with hysteresis. A finite difference solution with time was developed and calculations were made for the F-111 TACT airplane. Static pressure data for the F-111 TACT were used to determine the step forcing function due to SITES transition. However, no data were available for the hysteresis hence a parameter study was conducted allowing the hysteresis effect to vary. Very small hysteresis effects, which were within expected bounds, were required to obtain reasonable response levels that essentially agreed with flight test results. Also in agreement with wind tunnel tests, LCO calculations for the 1/6-scale F-111 TACT model showed that the model should not have experienced LCO.

- **The Role Of Shock Induced Trailing Edge Separation (SITES) In Limit Cycle Oscillations (LCO) Has Been Described**
- **The Flip-Flop Characteristic Of Transition To SITES and Its Hysteresis Has Been Shown To Be A Potential Source Of LCO**
- **A Very Simple Non-Linear Math Model Was Assembled and Solved With A Finite-Difference Approach**
- **The Math Model Used Static Pressure Model Data and Standard Flutter Solution Results; However, Hysteresis Data Were Not Available and Had To Be Assumed For This Study**
- **Calculations For The F-111 Tact Aircraft Showed Order Of Magnitude Agreement With Flight Test**
- **Calculations Also Showed That The 1/6-Scale Model Should Not Have Experienced LCO-Which Agrees With Observations**

Figure 12

REFERENCES

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3. Triebstein, H.: Steady and Unsteady Transonic Pressure Distributions on NACA 0012. Journal of Aircraft, vol. 23, no. 3, March 1986, pp. 213-219.